Three Dimensional Sensing by Digital Video Fringe Projection

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Thesis supervisors: A/Prof. Jiangtao Xi and Prof. Joe F. Chicharo
This thesis is dedicated to my family and friends.
Abstract

Fast, high precision and automated optical noncontact surface profile and shape measurement has been an extensively studied research area due to the diversity of potential application which extends to a variety of fields including but not limited to industrial monitoring, computer vision, virtual reality and medicine. Among others, structured light Fringe Projection approaches have proven to be one of the most promising techniques. Traditionally, the typical approach to Fringe Projection 3D sensing involves generating fringe images via interferometric procedures, however, more recent developments in the area of digital display have seen researchers adopting Digital Video Projection (DVP) technology for the task of fringe manufacture. The ongoing and extensive exploitation of DVP for Fringe Projection 3D sensing is derived from a number of key incentives the projection technology presents relative to the more traditional forms of projection. More specifically, DVP allows for the ability to accurately control various attributes of the projected fringe image at high speed in software, along with the capabilities to develop multi-channel techniques via colour projection. Furthermore, considering the typical DVP source is capable of projecting a standard 24 bit bitmap computer generated image, when interfaced to a personal computer, DVP makes for a very affordable projection source. However, despite the aforementioned incentives, in contrast to the more traditional methods of generating fringe images, the digitally projected fringe signal presents a number of shortcomings which ultimately hinder the effective application of the technology for Fringe Projection 3D sensing.

This thesis aims to improve the effectiveness of the deployment of DVP technology for Fringe Projection 3D sensing approaches. The proposed initiative is facilitated through extensive analysis of the application of DVP technology for fringe processing, and furthermore by the proposal of new digital fringe calibration procedures.

Firstly, this work demonstrates a comprehensive survey of current Fringe Projection 3D sensing approaches including an introductory review of the rudimentary notion of projecting fringes for 3D data acquisition. The survey also provides a thorough description of the evolution of the three major forms of fringe processing i.e. Fringe Phase Stepping,
Fourier Fringe analysis and Direct Detection.

The limitations of DVP for Fringe Projection are demonstrated through the development of a novel fringe phase emulation approach. The phase emulation approach is subsequently employed to establish empirical insight into the application of DVP technology for Fringe Projection. More specifically, the preliminary empirical analysis is used to test the veracity of the application of the two chief DVP technologies (Liquid Crystal Display, LCD and Digital Light Processing, DLP, Texas Instruments) for Fringe Projection. Through this study the camera / projector non-linear intensity response is shown to be the single most significant shortcoming inherent to DVP based Fringe Projection implementations.

Following the findings of the preliminary empirical analysis the influence of the Display Gamma attributes of the projection system is extensively investigated. The harmonic structure of a typical digitally projected fringe signal is examined and an approximate analysis framework proposed. The framework is subsequently utilised to form a set of equations defining the true $\gamma$ sensitivity of a range of highly exploited fringe processing techniques. The approximate analysis is later verified and the practical significance of the findings demonstrated. Through this study the true nature of the Display Gamma related phase measuring residual error is revealed.

With the aid of a verified framework, investigations into additional Display Gamma related Fringe Projection phenomena is undertaken. More specifically, the optimisation of digitally projected fringes by fringe parameter manipulation is demonstrated. The temporal nature of digitally projected fringe images is studied for the well exploited single shot Fourier Transform Profilometry technique and the digital fringe harmonic dependence on the projector optical modulation transfer function is revealed. Subsequently, the elimination of Display Gamma related Fringe Projection phase measuring residual error for phase stepping techniques by projector defocus optimisation is shown.

Finally, a novel digital fringe calibration approach ideal for minimum shot fringe processing techniques is proposed. The calibration procedure is centered on the application of Artificial Neural Networks (ANNs) to correct the non-linear intensity distortion associated with the camera / projector system. Unlike previously proposed gamma correction techniques, the neural fringe calibration technique requires no additional data acquisi-
tion with effective calibration requiring no more than a single cross-section of a reference fringe. The neural network fringe calibration approach is also shown to significantly outperform simple filter based techniques of similar computational complexity. Given the reduced data requirements for the neural approach its application for multi-channel fringe calibration is also considered.
Certification

I, Matthew John Baker, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Matthew John Baker
17th February 2008
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I would firstly like to convey my thanks to both my supervisors, Associate Professor Jiangtao Xi and Professor Joe Chicharo. Without their continual support, guidance and patience this research would not have been possible. I thank them both for providing me with freedom in my research and the opportunity to travel and present my research at international conferences.

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doubtedly never be able to repay the latter debts, however, I should be ok for the financial debts.... eventually.

Finally, a very special thank you to my son Khye. Khye’s very being is certainly without a doubt my greatest accomplishment and although he cannot read many of the words contained within this document watching him grow and learn how to has truly been an inspiration for the completion of this work.
List of Publications

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2. Matthew J. Baker, Joe Chicharo and Jiangtao Xi, “Accuracy Limitations in Profilometric Metrology Schemes using Digital Structured Light”, *Accepted subject to revisions, IEEE Transactions on Instrumentation and Measurement*

3. Matthew J. Baker, Jiangtao Xi and Joe Chicharo “The Implications of Display Gamma for Fringe Projection 3D sensing using Digital Video Projection”, *To be submitted to Applied Optics*

Conference publications:


tions, Boston, Massachusetts, USA, October 25-26, 2005, (EI: 06109749730).


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<tr>
<td>3D</td>
<td>Three-Dimensional</td>
</tr>
<tr>
<td>2DFTP</td>
<td>Two-Dimensional Fourier Transform Profilometry</td>
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<tr>
<td>Am-Si</td>
<td>Amorphous Silicon</td>
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<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
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<td>AOM</td>
<td>Acousto-Optic Modulator</td>
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<td>CCD</td>
<td>Charged Couple Device</td>
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<td>DC</td>
<td>Direct Component</td>
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<tr>
<td>DFP</td>
<td>Digital Fringe Projection</td>
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<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<td>DLP</td>
<td>Digital Light Processing</td>
</tr>
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<td>DMD</td>
<td>Digital Micromirror Device</td>
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<td>DPD</td>
<td>Direct Phase Detection</td>
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<td>DVP</td>
<td>Digital Video Projection</td>
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<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
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<tr>
<td>FTP</td>
<td>Fourier Transform Profilometry</td>
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<tr>
<td>I3PSP</td>
<td>Improved Three Step Phase Stepping Profilometry</td>
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<tr>
<td>IFTP</td>
<td>Improved Fourier Transform Profilometry</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>LCD</td>
<td>Liquid Crystal Display</td>
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<tr>
<td>LCOS</td>
<td>Liquid Crystal on Silicon</td>
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<tr>
<td>MEMS</td>
<td>MicroElectroMechanical System</td>
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<tr>
<td>MFTP</td>
<td>Modified Fourier Transform Profilometry</td>
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<td>MMP</td>
<td>Modulation Measurement Profilometry</td>
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<td>OPD</td>
<td>Optical Path Difference</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>PC</td>
<td>Personal Computer</td>
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<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
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<tr>
<td>PMP</td>
<td>Phase Measuring Profilometry</td>
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<tr>
<td>Poly-Si</td>
<td>Polycrystalline Silicon</td>
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<tr>
<td>PSD</td>
<td>Position Sensitive Detector</td>
</tr>
<tr>
<td>PSI</td>
<td>Phase Shifting Interferometry</td>
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<tr>
<td>PSP</td>
<td>Phase Stepping Profilometry</td>
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<tr>
<td>SLM</td>
<td>Spatial Light Modulator</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>SPD</td>
<td>Spatial Phase Detection</td>
</tr>
<tr>
<td>TFT</td>
<td>Thin Film Transistor</td>
</tr>
<tr>
<td>TI</td>
<td>Texas Instruments</td>
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<td>TN</td>
<td>Twisted Nematic</td>
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